ACTUARIAL SOCIETY OF SOUTH AFRICA

APN 110: ALLOWANCE FOR EMBEDDED INVESTMENT DERIVATIVES

Classification

This Advisory Practice Note (APN) provides guidance for members of the Actuarial Society of South Africa in reserving for embedded investment derivatives. APN 110 is applicable to all valuations with a valuation date on or after 31 December 2012 in respect of long-term insurers registered in South Africa. APN 110 replaces PGN 110. Where legislation or other documentation refers to PGN 110 it should be interpreted as APN 110.

Abstract

This APN recommends suitable methodology to be used by statutory actuaries in reserving for embedded investment derivatives. The APN recommends the minimum steps that should be taken by the actuary when setting up a reserve.

This APN recommends the use of market-consistent stochastic models to quantify reserves required to finance possible shortfalls in respect of embedded investment derivatives. The use of stochastic models does not necessarily imply the use of Monte-Carlo methods. In some cases, it is possible to quantify the extent of the investment guarantee reserve using closed-form methods.

Purpose

The purpose of this APN is to ensure that the actuary reserves adequately for embedded investment derivatives. This APN does not impose a new reserving requirement, as the actuary is already obliged to reserve for this particular liability in terms of SAP 104. However, this APN recommends that reserving for embedded investment derivatives is market-consistent.

Legislation or Authority

This APN emanates from the work and consultations of the Investment Guarantees Subcommittee of the Life Assurance Committee of the Actuarial Society of South Africa.

Application

Statutory actuaries who perform valuations as at 31 December 2012 or later in respect of long-term insurers registered in South Africa.

Author

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Status

Version 1	Effective for valuations performed as from 31 December 2003
Version 2	Effective for valuations performed as from 31 December 2007
Version 3	Effective for valuations performed as from 31 December 2008
Version 4	APN 110 effective for valuations performed as from 31 December 2012

DEFINITIONS

Actuary	A statutory actuary responsible for the valuation of long-term insurance liabilities in South Africa
Asset share	The fund account (for a single policy or group of policies), plus the share of the bonus stabilisation reserve, if applicable
CAR	Minimum statutory capital adequacy requirements described in SAP 104 issued by the Actuarial Society and in a Board Notice to the Long-term Insurance Act entitled "Prescribed requirements for the calculation of the value of the assets, liabilities and Capital Adequacy Requirement of long-term insurers"
CTE	Conditional Tail Expectations
Embedded investment derivative	An investment derivative, including those as defined in IAS39, underlying a policy contract. These include derivatives offering contractual minimum benefits on the underlying policy contract, but which allow the policyholder contractual or discretionary participation in any upside above the contractual minimum benefit. The contractual minimum benefits could include, but are not limited to, maturity or surrender benefits, bonus rates or annuity rates.
EV	Embedded value calculated in accordance with APN 107 issued by the Actuarial Society
Guarantee reserve	Reserve required to meet the expected cost of the embedded investment derivative
Market-consistent stochastic model	A model that reproduces the market prices of tradable assets as closely as possible (See Appendix 2)
Real-world stochastic model	A model that projects investment returns for various asset classes according to the estimated probability distribution based on historical observations and future expectations (See Appendix 2)
Risk-neutral stochastic model	A model that projects investment returns for various asset classes where assumed risk premiums are equal to zero
Stochastic investment return models	A model projecting future investment returns using probabilistic methods

1. SCOPE

This APN recommends suitable methodology to be used by actuaries to reserve for embedded investment derivatives. The APN recommends the minimum steps that should be taken by the actuary.

While this APN focuses on the calculation of reserves required for minimum maturity guarantees, it is recommended that the methods described in this APN should also be used to quantify the liability in respect of all embedded investment derivatives.

Embedded investment derivatives that are covered by this APN include:

- Minimum investment maturity guarantees
- Guaranteed annuity options
- Minimum investment related death or other risk benefits
- Minimum investment related surrender benefits
- Minimum increase rate guarantees on variable annuities
- Implied investment guarantees related to conventional with-profit and smoothed bonus business in the form of vested/guaranteed bonuses
- Explicit or implicit minimum investment return guarantees on universal life policies' fund accounts (e.g. a guarantee term on a with cover universal life policy implies a guaranteed fund value and surrender value of zero on the guarantee expiry date).

It should be noted that the above list is not all-encompassing and that there might be other embedded investment derivatives to which the APN applies.

This APN recommends the use of market-consistent stochastic models to quantify reserves required to finance possible shortfalls in respect of embedded investment derivatives. Where there are no traded market instruments from which to calibrate the market-consistent model, the actuary may apply alternative methods and judgement provided that he/she can argue that such derived values used to calibrate the model are probable in the market.

The use of stochastic models does not necessarily imply the use of Monte-Carlo methods. In some cases, it is possible to quantify the extent of the investment guarantee reserve using closed-form methods.

2. INTRODUCTION

Many South African long-term insurers have in the past and are currently writing individual life policies with embedded investment derivatives. An example of such a policy is an individual life policy where the benefit on death or maturity is dependent on the investment performance of the underlying assets (e.g. smoothed-bonus or market-related policies), but with a minimum contractual annual investment return guarantee (e.g. 4% per annum) at the date of the claim.

Prior to 2003, there had been no explicit professional guidance to South African actuaries on how to reserve specifically for these embedded investment derivatives. Because inflation was high in the past relative to the guaranteed investment return underlying some of these derivatives, the derivative was not deemed particularly onerous and the issue therefore did not receive much attention. However, longer-term expectations for future inflation in South Africa are now at a level where the potential costs of these derivatives are no longer immaterial. A need arose for professional guidelines recommending a scientific reserving basis for embedded investment derivatives underwritten by long-term insurers registered in South Africa.

In 2003, the Maturity Guarantees Subcommittee of the Life Assurance Committee of the Actuarial Society produced Version 1.0 of this APN. PGN 110 version 1 focused on minimum maturity guarantees. Reference was, however, also made to other benefits arising from minimum investment return guarantees. Version 1 also focused on real-world stochastic models, although market-consistent models (risk-neutral and deflator-based) were suggested as a possible alternative.

Version 2 of the APN was issued in the beginning of 2007 recommending the use of market-consistent models for determining the liability arising from all embedded investment derivatives. Real-world simulation models were still recommended for calculating the capital adequacy requirements in respect of embedded derivatives. The CAR calculation was based on conditional tail expectations (CTE's) of the simulated discounted guarantee shortfall at the maturity date of the policy.

Version 3 of the APN was issued in April 2008 and built on Version 2. The use of market-consistent models was still recommended. However, the CAR calculation was brought in line with the resilience CAR calculation methodology discussed in SAP 104.

Version 4 renamed PGN 110 as APN 110 in late 2012.

3. METHODOLOGY

Deterministic actuarial valuation techniques based on best estimate assumptions use expected investment returns that might not appropriately allow for the complexities of the investment returns implied by embedded investment derivatives. For example, when reserving for minimum maturity guarantees, the deterministic return assumed in the valuation is often higher than the guaranteed minimum investment return. Such deterministic methods are thus not appropriate to quantify the reserves that must be held to fund possible shortfalls under embedded investment derivatives.

In order to quantify the reserves, stochastically simulated future economic variables are used to project the liabilities arising from embedded investment derivatives. The additional liabilities (i.e. shortfalls) at claim date are then discounted at an appropriate discount rate to determine the present value of the reserve required at the valuation date.

4. STOCHASTIC INVESTMENT RETURN MODEL

4.1 Type of stochastic model

No specific investment return projection model is prescribed. The actuary may use any market-consistent stochastic investment return projection model that he/she deems appropriate for purposes of quantifying reserves required to meet the potential cost of embedded investment derivatives.

4.2 Assumptions and parameters

Each stochastic model will have its own unique set of parameters. Most models will however have as inputs a term structure of interest rates as well as parameters relating to the variability of future investment returns on the assets backing the policies under the guarantee.

It is recommended that, for valuation of market-consistent liabilities, a zero coupon yield curve, either stripped from an average of mid-swap curves of leading participants in the local currency (ZAR) interest rate swaps market, or based on the zero coupon government bond curve, be used. Volatilities of the stochastically simulated investment returns should be in line with those implied by tradable derivatives with appropriate underlying assets.

In order to ensure that reserving is market-consistent, parameters of the model used to value the embedded derivative should be determined using market prices of tradable derivatives related closely to the embedded derivative in question. It is recognised that due to the long-term nature of insurance contracts, tradable derivatives in the underlying assets may not be available for certain maturities. Therefore, to determine the assumptions underlying the stochastic model, a combination of historical volatility analysis and solving for volatilities implied by derivative prices may be required.

To determine the relevant volatility assumptions underlying the stochastic model using historical analyses, a pragmatic approach is required. For example, the realised market price volatility of the underlying asset (over an acceptable historic period) could be calculated for a term consistent with each required volatility parameter. This calculated realised volatility can then be compared to the implied volatility parameter derived from derivative prices for terms where tradable derivatives are available. This relationship can then be extrapolated, together with the calculated realised volatility, to determine the volatility parameter for terms where tradable derivatives are not available. The volatility parameter may be time-dependent and may itself be a stochastic process.

4.3 Expected returns

Expected returns on the assets underlying the reserve should not, in theory, affect the reserve in respect of an investment return guarantee. For example, if a reserving exercise for a maturity guarantee on a unit-linked policy is carried out, the liability should not be affected by the expected return assumption. A higher expected return assumption will be exactly offset by the state-price deflators used for discounting the additional liabilities (See Appendix 3 for a brief discussion of state-price deflators).

4.4 Mean-reverting returns

In the case where an actuary is using a mean-reverting model for returns on certain asset classes, the actuary must be satisfied that the principle of market-consistency is not violated and that the model still provides good approximation for prices of traded derivatives. Academic literature (e.g. Smith (1996), Huber (1997), Maitland (1996)) on the subject of stochastic models of equity returns points out dangers of mean-reverting returns for risky assets. If the expected return on an asset class at any time is conditional on the dividend yield or earnings yield of that asset class, this may affect the reserve in respect of an investment return guarantee if such yields are mean-reverting. For example, if the price of the underlying asset increases as a result of a fall in yields and yields are mean reverting, a dynamic hedge that increases its holding in the underlying asset as prices rise, and vice versa, will cost more because of its inverse exposure to the underlying. In the absence of arbitrage, the cost of the investment return guarantee is the cost of the replicating portfolio.

4.5 Number of iterations

If a simulation exercise is carried out, the actuary must decide on a practical number of iterations of future asset price scenarios. The recommended minimum number of required iterations is 1000. Preferably at least 2000 iterations should be performed.

Where variance reduction techniques are used, or where there is convergence of the iterations of the specific stochastic model, fewer iterations may be performed. The actuary should however use his/her judgement in this case.

5. RESERVING METHODOLOGY

Embedded investment derivatives other than those of a minimum investment return at maturity are often marketed by life offices (e.g. annuity option guarantees, minimum increase rate guarantees on variable annuities). All these, as well as maturity guarantees, are essentially investment derivative instruments embedded in life insurance policies.

5.1 Calculating reserves for maturity guarantees

This section focuses on the reserving calculations that would apply to maturity guarantees. The actuary should however apply similar methodology to all other applicable embedded investment derivatives.

For each policy or group of policies with an applicable minimum contractual maturity value, the market value of the underlying assets (i.e. the asset share) as at the valuation date is used as the starting point. This value is accumulated with future premiums at the stochastically simulated investment returns allowing for charges and taxation to determine the projected maturity value for each policy or group of policies. The projected maturity values are calculated based on best estimates of all future contingencies (e.g. premium increases), other than decrements and the future investment returns.

For the purposes of this APN, the decrements should be divided into "final-off" decrements and "partial-off" decrements. The former are the decrements that lead to a policy being removed from a life office's in-force file (such as deaths, lapses or surrenders). The latter are the decrements such as part-surrenders and paid up conversions, which affect the build-up of the value of units but do not lead to a policy being removed from the in-force file. Allowance for partial-off decrements in the build-up of fund values and guaranteed values will depend on the nature of the guarantee offered and will therefore differ between companies. The actuary must be satisfied that an appropriate allowance is made for partial-off decrements in determining the investment guarantee liability.

For each policy or group of policies, the projected maturity value is compared to the contractual minimum guaranteed maturity value, where the contractual minimum guaranteed maturity value is also calculated without allowance for decrements. If the projected maturity value exceeds the guaranteed maturity value, a zero shortfall is recorded. If the projected maturity value is less than the minimum guaranteed maturity value, the shortfall should be reserved for. The actuary may take credit for the accumulated value of premiums explicitly charged for and expected to be reserved for the minimum maturity guarantees (i.e. if the projected charges for a particular simulation exceed the projected shortfall, a surplus may be recorded, to the extent that these projected charges are not offset against other liabilities or released in the embedded value).

The above methodology applies for smoothed bonus or market-related policies where the policyholder will be paid the greater of the projected maturity value and a guaranteed maturity value.

The shortfall or surplus at the maturity date must be discounted to quantify the value of the required reserve at the valuation date. The appropriate discount rate is discussed in Section 5.5 below. The discounted shortfall must then be multiplied by the probability of the policy (or group of policies) reaching maturity, by taking account of decrements. Once all the policies or groups of policies have been projected on a specific series of simulated investment returns, the entire process is repeated for each simulation of future investment returns. The average discounted shortfall across all simulations is then taken to be the guarantee reserve.

5.2 Other embedded investment derivatives

Although this APN lists a number of embedded investment derivatives, it is important that the reserving methodology be applied to each type of embedded investment derivative. The reserve should be calculated as the expected value of the discounted payoff from the embedded investment derivative.

Some guarantees offered by life offices could be very complex instruments. As such, they may be very difficult to model precisely. Parameter estimation may often also be problematic. The actuary needs to bear in mind that the appropriate recognition of the nature and extent of risk involved in those guarantees is more important than surgical precision in the valuation models. For this reason, the actuary must use his/her judgement to strike an appropriate balance between complexity and practicality.

5.3 Smoothed bonus business

For contracts where benefit payments are smoothed over time, existing policyholders are expected to subsidise benefit payments when funding levels are low and vice versa. Provision for the full shortfall between the guaranteed minimum maturity value (or other investment guarantee that may apply) and the value of the underlying assets may therefore not necessarily be consistent with the way in which the fund is managed in practice. The actuary may allow for this internal subsidy mechanism between generations of smoothed bonus policyholders. However, it is unlikely that the internal subsidy arrangement will remove the need for a reserve in total and the actuary must still reserve for those losses that could potentially be incurred by shareholders.

Starting with the asset share at the valuation date, the projected asset share will be calculated using simulated investment returns and assumptions about the future premium pattern. The projection of the value of the underlying assets should therefore allow for the investment policy in respect of policyholder funds.

Next, the policyholder liabilities should be projected by applying the bonus policy or rules in force at the calculation effective date. The assumed bonus distribution rules will determine the bonus rates that will be applied to the contract in each economic scenario simulated. These bonus rules should be consistent with the wording of the life office's Principles and Practices of Financial Management (PPFM). The policyholder liabilities should thus be projected dynamically in conjunction with the asset share.

The guaranteed minimum maturity values (or other investment guarantees that may apply) should also be projected. These guaranteed or minimum values should allow for future vested bonuses as well as the future accrual of the guaranteed values at the underlying guaranteed rate (where applicable).

The excess of the guaranteed minimum maturity values (or other investment guarantees that may apply) over the projected policyholder liabilities (based on projected bonuses), constitutes a future liability that must be reserved for.

The actuary should also bear in mind that a deterioration in funding level (i.e. the difference between the projected asset share and the value of policyholder liabilities based on projected bonuses) itself may constitute a guarantee cost where available assets are insufficient to support benefit payments (including declared bonuses) going forward. This cost should also be quantified as part of the investment guarantee reserve calculation.

Wherever applicable, allowance for offsetting management actions may be made in calculating the potential losses to shareholders. This includes the reduction or removal of bonuses accruing to future generations of policyholders. Credit for the internal subsidy mechanism and the management action offsets may be taken only where these have been resolved by the Board and where the statutory actuary is satisfied that:

- In practice, the office would allow the modelled cross-subsidies and apply the modelled management actions. Special consideration should be given to any cross-subsidies between smoothed bonus policyholders that enjoy different levels and types of investment guarantees.

- Such cross-subsidies and management actions are not contrary to any representations made to the policyholder (including marketing literature and PPFM) that may have impacted policyholder reasonable expectations.

To the extent that the cost of the investment guarantee is borne by shareholders, the investment guarantee reserve may not be used to reduce the bonus stabilisation reserve.

5.4 Reversionary bonus business

With reversionary bonus business, life offices often have discretion in terms of bonus distribution as well as investment policy. The extent of the discretion and the way in which it is applied will have a direct impact on the cost of any minimum investment return guarantee. For example, aggressive profit distribution through reversionary bonuses early on in the life of a policy will increase the guaranteed sum assured earlier and the cost of the guarantee will therefore be greater than in a situation where the majority of the profit distribution is done via a terminal bonus. Also, where a life office has discretion to vary the investment policy of the assets underlying the asset share (for example, if the office can match the guaranteed sum assured with government bonds), the guarantee reserve will be lower.

Application of discretion regarding the distribution of profits and changes in investment policy should be allowed for in the calculation of the reserves and capital requirements in respect of minimum investment guarantees. The way in which discretion is assumed to be applied should be in line with current bonus philosophy and reasonable expectations of policyholders as well as any existing documentation codifying the Principles and Practices of Financial Management (PPFM).

The calculation of minimum investment guarantees in respect of reversionary bonus business should follow similar principles to those described in the section on smoothed bonus business above. As an example, consider a with-profits endowment policy. At any point in time, the initial sum assured together with any bonuses vested so far will constitute a minimum guaranteed maturity value. Starting with the asset share at the valuation date, the projected asset share will be calculated using simulated investment returns and assumptions about the future premium pattern. The assumed bonus distribution rules will help determine the reversionary bonus rates that will be applied to the contract in each economic scenario simulated. If the projected asset share at maturity in a particular economic scenario exceeds the initial sum assured plus attaching bonuses (both existing at the valuation date and projected on model assumptions), no shortfall is recorded and surplus (or part thereof) can be distributed as a terminal bonus. If the projected asset share is below the initial sum assured and vested bonuses, a shortfall for the particular simulation is recorded and discounted as described in Section 5.5 below.

5.5 Appropriate discount rate

The shortfalls (and if applicable, surpluses) at maturity, if any, must be discounted to the valuation date to quantify the reserves required to meet the investment return guarantees. If investment returns are simulated under the risk-neutral probability measure (i.e. if the expected return on each asset class is the risk-free rate), the appropriate discount rate for each projection interval is the simulated risk-free rate of return for that interval. If the expected return is assumed to be different from the risk-free rate, the appropriate discount factor is the state-price deflator for the particular simulation. The risk-free discount factor is a special case of a state-price deflator where the risk premium on each asset class is zero.

5.6 Taxation

Allowance for taxation of linked assets (in case of unit-linked business) or asset share (in case of smoothed bonus or reversionary bonus business) can be made at the actual tax rates applicable to the relevant asset classes, taking into account the tax fund in which the corresponding liabilities are held. Allowance for tax on the assets backing the guarantee reserve is however more complex (as described below).

In order to properly calculate the market-consistent value of a contingent liability, one would need to model the composition of the hedging portfolio at each projection point in the simulation and calculate tax payable at each point in time on the investment return on the hedging portfolio. This potentially involves nested simulations and run-times will make the calculation impractical. An alternative pragmatic approach (although not entirely theoretically correct) is as follows:

1. First, calculate the market-consistent value of the liability ignoring tax on assets backing the guarantee liability (i.e. risk-neutral discount factors or deflators should be used without adjusting for tax). Denote this result by A.
2. Next, calculate the real-world CTE(0) reserve also ignoring tax on investment guarantee reserve (See Appendix 1 for a discussion of Conditional Tail Expectations (CTE's)). Denote the result by B.
3. Calculate the real-world CTE(0) reserve, but assume that income and capital gains on the investment guarantee reserve is incurred at the applicable rates. Denote the result by C.
4. Calculate the market-consistent reserve allowing for tax as R = (A x C) / B. In other words, assume impact of tax is proportionally the same for the market-consistent and real-world reserve.

The approach described above might require significant time and resources, since a real-world reserve needs to be calculated. However, this reflects one approach and other pragmatic approaches would also be acceptable.

5.7 Other assumptions

The reserves discussed in this APN should be calculated based on best estimates of all future contingencies other than investment returns. Unless stated otherwise, the assumptions should be consistent with the best estimates used in the valuation. The actuary may make allowance for the possible interaction between future decrements and the future investment returns. For example, the actuary may assume reduced lapse and surrender experience if the particular iteration projects particularly poor investment returns that renders the minimum maturity guarantee more valuable. Automatic premium increases must be taken into account based on a realistic take-up rate.

5.8 Margins

The guarantee reserve calculated on a market-consistent basis should not include compulsory margins. However, in certain circumstances, the actuary may consider it appropriate to include discretionary margins.

6. CAR CALCULATION

The size of the liability arising as a result of investment guarantees and other embedded derivatives is likely to be very sensitive to adverse economic scenarios such as a substantial reduction in the level of the equity market or a significant change in the term structure of interest rates. The minimum amount of assets the insurer needs to hold in excess of its liabilities to ensure its solvency in adverse circumstances should therefore take into account the effect of the existence of embedded derivatives.

The effect of embedded derivatives on the size of OCAR (as defined in SAP 104) should be quantified by including the liabilities in respect of embedded derivatives in the calculation of the g(i) item of IOCAR (as defined in SAP 104). This implies a re-calculation of the liability in respect of embedded investment derivatives assuming changes in the values of assets and economic variables (such as interest rates) as specified in SAP 104. The assets backing the liabilities in respect of investment derivatives should be re-valued accordingly. Management action considerations spelt out in SAP 104 should be taken into account where applicable.

It should be noted that the above requirement implies that the stochastic model should be re-calibrated in order to be consistent with the shock economic conditions in a resilience test scenario contemplated in the item g(i) of IOCAR. This re-calibration should involve a change in the initial zero-coupon yield curve reflecting a 25% proportional increase or reduction in zero-coupon bond yields at all durations (whichever is more onerous to the total IOCAR).

It is not required that any other parameters (such as risk premia, volatilities or correlations) of the stochastic model should be changed. The simulation of future investment returns and discount factors for the purposes of the CAR calculation should be consistent with this new calibration.

SAP 104 also requires re-valuation of liabilities on a worse investment return scenario. In the context of investment guarantees, the worse investment return scenario contemplated in SAP 104 can be interpreted as a proportional 15% reduction in the risk-free zero-coupon bond yields at all durations (with all other model parameters remaining unchanged). While it is very likely that the resilience scenario will be more onerous than the worse investment return scenario, the actuary must consider which scenario will be more onerous for the total CAR for a specific company.

7. OTHER CONSIDERATIONS

7.1 Grouping of policies

It may be impractical to apply the Monte Carlo investment return projections on a policy-by-policy basis. The actuary can make use of a representative sample of the relevant policy book. The actuary must be satisfied that the sample is appropriate and representative of the policy book concerned.

7.2 Timing of the calculation of the reserve

The ideal approach is to calculate the reserves and CAR described in this APN at the financial year-end or half-year end. The actual market value of the underlying assets (i.e. the asset shares) as at the valuation date should be used as the starting values to project the expected asset shares at the claim date. The reserves required to meet the embedded investment derivatives and CAR may be calculated prior to the valuation date, since there may not be sufficient time during the main valuation process. However, in this case, the necessary adjustments based on changes in the values of the underlying assets as well as changes in the economic and market conditions between the calculation date and valuation date, must be estimated. The actuary should use his/her judgement regarding the adjustment required to determine the reserves and CAR at the valuation date since the theoretical calculation might be too complex and time-consuming.

8. DISCLOSURES

The actuary must use the market-consistent stochastic model to price the following contracts and disclose these prices as part of the information required under this APN in the statutory actuary's report required in terms of APN 103. These prices should aim to replicate market prices (and as such include the profits that banks would load into their expected volatilities). Please note that the FTSE/JSE TOP40 referred to in this section is a capital return, as opposed to a total return, index, whereas the ALBI is a total return index.

8.1 State whether the actuary used closed-form methods (as opposed to Monte Carlo simulation techniques) to quantify the liability in respect of embedded investment derivatives.

8.2 Prices and implied volatilities on the following put options on FTSE/JSE TOP40 index:

Maturity	**Strike**
1 year	Spot
1 year	0.8*Spot
1 year	Forward
5 years	Spot
5 years	(1.04^5)*Spot
5 years	Forward
20 years	Spot
20 years	(1.04^20)*Spot
20 years	Forward

Where:

Spot refers to the price of the equity index at the valuation date;

Forward = *Spot* $* e^{(r-q)T}$;

T is the term to maturity of the option;

r is the risk-free interest rate for maturity at time *T*; and

q is the expected dividend yield on the index over the term of the option.

The dividend yield q used in the calculation of the strike price should be consistent with the expected dividend yield on the equity index implied by the calibration of the stochastic model.

8.3 A 5-year put with a strike price equal to $(1.04)^5$ of spot, on an underlying index constructed as 60% FTSE/JSE TOP40 and 40% ALBI, with rebalancing of the underlying index back to these weights taking place annually.

8.4 A 20-year put option based on an interest rate with a strike equal to the present 5-year forward rate as at maturity of the put option (stripped from the zero coupon yield curve derived in Section 4.2 above), which pays out if the 5-year interest rate at the time of maturity (in 20 years) is lower than this strike. The payoff will be calculated as

Max{Strike – simulated 5-year interest rate at time 20 years, 0}.

The payoff of the above option should be assumed to occur at time 20 years.

8.5 The zero coupon yield curve (derived in Section 4.2 above), used in the asset projection. The yields should be disclosed for years 1 – 5 and in 5-yearly intervals thereafter.

8.6 Where the stochastic model used in the calculations was calibrated at a date other than the reporting date.

8.7 If the actuary opted to calculate the reserves at a different date, the effective date should be disclosed.

REFERENCES

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Maitland, A.J. (1996). *A Review of Thomson's Stochastic Investment Model.* Transactions of the Actuarial Society of South Africa, 1996.

Smith, AD (1996). *How actuaries can use financial economics.* BAJ. **2**, V, 1057-1193.

APPENDIX 1: Conditional Tail Expectations (CTE)

The 100p% Conditional Tail Expectation or CTE(p) (0<=p<=1) is the expected value of a random variable given that the value is greater than V_p (where V_p is the 100*p^{th} percentile). CTE(p) is calculated as the arithmetic mean of the highest 100(1-p)% reserves from the simulation. For example, CTE(0.6) would be the average of the highest 40% of the reserves (therefore assume that the reserve will definitely be larger than the 60^{th} percentile). Now take the arithmetic average of all the reserves greater than the 60^{th} percentile to find the expected value of the reserves under this assumption.

If p = 0, CTE(p) = E[X] = mean, because the whole population is now being considered. This means that for any p > 0, CTE(p) must be greater than (or equal to) the mean. This overcomes one of the disadvantages of using percentiles.

Hardy (2001), gives the following advantage of using CTE's:

"The CTE is easy to calculate using the simulation output, as CTE(p) is the mean of the highest cost 100(1-p)% outcomes from the simulation. This is very simple to implement and understand. By taking an average of the worst case projections, the estimate is more robust with respect to sampling error than the quantile method." Consequently CTE's shouldn't differ as much as the estimates of the higher order percentiles from one set of simulations to the next.

The report by the Canadian Institute of Actuaries entitled "CIA Task Force on Segregated Fund Investment Guarantees" (August 2000) states: "The CTE approach provides a more stable result than simply selecting a 'percentage of scenarios' coverage approach (i.e. percentiles). This is because the CTE measure uses an average of all scenario results beyond the selected point, while the percentile approach by definition selects a single scenario to establish amounts."

APPENDIX 2: Market-consistent and real-world models

The following section provides an overview of real-world and market-consistent asset models.

1. Real-world models

Real-world stochastic models simulate future values of economic variables such as inflation, equity returns, etc. according to a probability distribution observed in the real world (usually with parameters estimated from historical data). Well-known examples of real-world models include the Wilkie model and the Thomson model.

One possible shortcoming of the real-world models is that these models generally take credit for risk premiums for the risky asset classes. Hence, the model might not be arbitrage-free (in some circumstances). Under the no-arbitrage principle, all asset classes produce the same risk-adjusted return. Therefore, under the no-arbitrage principle, the assets backing the investment guarantee reserve should not have an impact on the size of the reserve.

Another problem is that the assumptions underlying the real-world models are often estimated from historical data, which may not be a good guide to possible future values of the economic variables concerned.

It must be pointed out that real-world and market-consistent models are not mutually exclusive and that it is possible for a real-world model to be market-consistent. However, although real-world models could in theory be market-consistent, the methods applied in practice to derive the assumptions, together with the manner in which these models are applied, often result in the models not being market-consistent.

2. Market-consistent models

Market-consistent simulation models produce prices for assets and liabilities that are directly verifiable from the market. These models also generate scenarios for future values of economic variables but do so based on assumptions that are derived from actual market prices of tradable assets.

Market-consistent simulation models can be either risk-neutral or deflator-based. Risk-neutral and deflator-based models are discussed in more detail in appendix 3 below.

Market-consistent models have the advantage that they generally produce arbitrage-free returns. Arbitrage-free models ensure that the value placed on the investment guarantee reserve is unique and independent of the assets in which it is invested (which again is a more plausible result in a fair-value framework).

A further advantage of market-consistent techniques is their objectivity (in theory). This is achieved by using assumptions that are derived from the market prices of tradable assets.

The market-consistent approach is therefore more consistent with the fair value environment as it places values on liabilities that reflect the price they would be traded at if a liquid market were to exist.

APPENDIX 3: State-price deflators and risk-neutral models

1. What are deflators?

A deflator operates as a stochastic discount factor where the discount rate is dependent on the state (and time period).

When calculating deflators, it is assumed that, according to the law of one price under the arbitrage free principle, two instruments that generate the same cash flows under the same circumstances at the same time must have the same price.

Theoretically it is possible to determine the value of an instrument (any set of cash flows) by constructing a portfolio of assets (for which the price can be observed in the market), in such a way that it has the same cash flows as the instrument in each possible economic state and time period. The price of the instrument is then the price of the portfolio of assets.

This logic is used to determine the current price of arbitrary state price securities that pay out one in a specific economic state and zero in all other states. These state prices can be used to value any instrument by multiplying the state price to the amount of the cash flow in each state. The state price is equal to the deflator (or stochastic discount factor) multiplied by the probability of the state occurring.

In a simplified scenario where we only have two states (say low and high), and the high and low state have equal probability to occur, the deflators for a set of state prices would be:

	State price	**Probability**	**Deflator**
High state	0.35	0.5	0.7
Low state	0.6	0.5	1.2

These deflators can be used to determine the value of any cash flow simply by multiplying the deflator for each state with the cash flow for that state and taking the average. The same set of deflators can be used, regardless of the cash flow we are trying to value or the type of assets used to back the cash flow.

2. Risk-neutral models

Risk-neutral models assume that expected investment returns on all asset classes are equal to the risk-free rate and discount factors are similarly based on the risk-free rate. Under the risk-neutral method, the real-world probabilities are modified into risk-neutral probabilities. The price of a security is then determined as the expected (based on risk-neutral probabilities) discounted payoff where the discount rate is the simulated risk-free rate of return.